REPORT

on the research carried out during Laurence Carassus' visit in the framework of the Centre of Excellence programme

Following our proposed schedule of research, we concentrated our efforts mainly on two problems of arbitrage theory : stationary markets and large financial markets.

The first of these problems has been initiated in former works of Laurence Carassus and others ([1, 2]). The basic problem consists in finding an appropriate formulation for the intuitive concept of a stationary market: i.e. a market where investment opportunities are available at any date and in any state of the world at the same conditions as should be the case in an economy in equilibrium.

The case of deterministic cash flows was solved in [1]. In [2] a binomial tree model was considered. So far this is the only stochastic model where a satisfactory problem formulation and results are available. We tried to approach the notion of stationarity in another way, by introducing an underlying stationary process describing the present state of the economy. We investigated the corresponding functional analytic framework and isolated a key assumption for obtaining meaningful results. Then we had a close look on how this assumption is verified or violated in concrete models. We showed that, even in fairly simple models the assumtion *fails*, hence new technical tools are to be found for settling related conjectures (or eventually refute them). These directions form an object of further research.

Our second main domain of interest concerns lage financial markets. These are used to model situations where a plethora of investment possibilities are at the agent's disposition (just like in today's global market). Such models allow for risk diversification, i.e. reducing or even eliminating risk by trading in a large number of securities. As in statistics, where properties of large samples are treated in a mathematical framework involving an infinite sequence of experiments, large financial markets are supposed to have a countable number of assets and we are interested in their "asymptotic" properties. In particular, basic questions are investigating the existence of asymptotic arbitrage, defining and comparing different notions of arbitrage and relate absence of arbitrage to the existence problem of "fair" prices.

While these problems are quite well understood in market models with finitely many assets, in large markets one faces considerable difficulties even in the simplest possible model. We were trying to compare existing definitions of asymptotic arbitrage from the literature. We presented examples and

counterexamples where these hypotheses hold (or do not hold). The model we focused on was an infinite sequence of independent coin tossings. We looked for explicit (and hence empirically testable) conditions on market parameters. These investigations broadened those of Miklós Rásonyi (see [3]) and others.

Our common work contributed to a deeper understanding of the machinery used in this context and also shed light on certain limitations of existing methods. Some new techniques also emerged which will hopefully bridge the gap which separates us from settling our conjectures. A collection of previously unknown examples and counterexamples was created.

Most of the work was carried out in the course of brainstorm-like joint work at the blackboard. Ideas were than clarified and put down in hand-written as well as electronic form - a manuscript note indicating the most important mathematical points is enclosed.

Laurence Carassus also gave a talk entitled "Explicit computation of the superreplication price in a model with discrete-time rebalancing". The talk exhibited a by now classical chapter in the pricing of derivative securities, then related results of the author and others were discussed in detail. The highlight of the lecture is the possibility of calculating the so- called "super-hedging price", whose theoretical description is well-known, but no closed-form formulae were avalable so far. The moral of these explicit result is that in certain models the superhedging price is somewhat too high, while in another class of models we find reasonable prices which are, in addition, easy to compute explicitly, using binomial trees only. The lecture constituted a sort of advanced continuation of the financial mathematics seminar of the Stochastic Systems Group last semester: the object of the present talk fitted well the series of topics treated there.

This cooperation enhanced international relationships between the Stochastic Systems Group of the Computer and Automation Institute of the Hungarian Academy of Sciences and the Laboratoire de Probabilité et Modèles Aléatoires, Universités Paris 6 et 7 (which is one of the largest Probability centres in the world). The collaborating fellows found that their research fields as well as their mathematical background and taste fit well and promise a long range fruitful continuation. A one month visit of Miklós Rásonyi is already planned for next year in LPMA. We hope to find financial support for a second visit of Laurence Carassus in MTA SZTAKI. There are a relatively small number of researchers working on discrete time models, in spite of the fact that these models seem to fit better reality. Hence the meeting of these two researchers is important for future research prospects.

References

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[2] Carassus, L. and Jouini, E. A discrete stochastic model for investment with an application to the transaction costs case. *Journal of mathematical Economics*, **33**, 57–80, 2000.

[3] Raásonyi, M. Equivalent martingale measures for large financial markets in discrete time. *Mathematical Methods of Operations Research*, forthcoming in 2003.

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